Description of reproduction numbers $R$

Reproduction number $R$ is a measure to describe the status of an epidemic dynamics. It depends on the virus’ infection properties and the social interactions in the population. There are different approaches to estimate this reproduction number that also depends on the status of the epidemics:

- **$R_0$:** At the beginning of an epidemics the $R_0$ value describes how many other people an infected person will infect on average. At the beginning of an epidemics this typically refers to the exponential increase of infected people. At the beginning of the COVID-19 epidemic this $R_0$ value ranged between 3.4 and 4.2 in Luxembourg. As only little data is available at the beginning of an epidemic, the $R_0$ value is generally only an estimate with big uncertainties.

- **$R_t$:** During the epidemics the dynamics changes, one the one hand due to potential measures that were put in place to reduce social interactions and on the other hand due to the increased percentage of immune people in the population. The $R_t$ value therefore describes the reproduction number at a given point in time. A pure $R_t$ value will reflect the potential spreading based on the social interactions. It however does not take into account the immunization rate within the population.

![Figure 1: Estimated $R_t$ values for the COVID-19 epidemic in Luxembourg.](image)

This shown $R_t$ value (Fig. 1) is the ratio of parameters beta and gamma in the SIR differential equation model, for which a daily estimate is obtained using the Extended
Kalman Filter. In the beginning of the epidemic, it coincides with the basic reproduction number, but differs at later stages. This coefficient reflects the effects of policies and people’s attitudes on the epidemic spread, and — in contrast with the basic reproduction number — it should remain unchanged, if these conditions do not change.

- $R_{\text{eff}}$: The effective average number $R_{\text{eff}}$ states how many people are infected by an infected person. To this end, the $R_t$ value has to be subsequently scaled by the immunization rate within the population (Fig. 1).

The effective reproduction number $R_{\text{eff}}$ can be also directly estimated from number of detected cases (Fig. 2). These estimates have typically a higher uncertainty and are more sensitive to fluctuating data.

Details for these calculations can be found here: [https://github.com/ResearchLuxembourg/covid-19_reproductionNumber/blob/master/src/estimation_R_eff.ipynb](https://github.com/ResearchLuxembourg/covid-19_reproductionNumber/blob/master/src/estimation_R_eff.ipynb)

![Figure 2: Effective reproduction number estimated by the approach of the London School of Hygiene & Tropical Medicine. Data points represent the average and light blue area the 50% confidence interval.](https://github.com/ResearchLuxembourg/covid-19_reproductionNumber/blob/master/src/estimation_R_eff.ipynb)

All these values represent estimates and obey some delay meaning that reported $R$ values represent the status of 5 to 10 days ago. This is caused by the incubation period of several days and delayed testing amongst others. To minimize these limitations and to provide a more accurate “nowcast” it would be essential to provide the date when the sample was taken and not the date of the test result.